SCOPE, SEQUENCE, and COORDINATION

A National Curriculum Project for High School Science Education

This project was funded in part by the National Science Foundation. Opinions expressed are those of the authors and not necessarily those of the Foundation.

SCOPE, SEQUENCE, and COORDINATION

SS&C Research and Development Center
Gerry Wheeler, *Principal Investigator*
Erma M. Anderson, *Project Director*
Nancy Erwin, *Project Editor*
Rick McGolerick, *Project Coordinator*
Arlington, Va., 703.312.9256

Evaluation Center
Frances Lawrenz, *Center Director*
Doug Huffman, *Associate Director*
Wayne Welch, *Consultant*
University of Minnesota, 612.625.2046

Houston SS&C Materials Development and Coordination Center
Linda W. Crow, *Center Director*
Godrej H. Sethna, *School Coordinator*
University of Houston-Downtown, 713.221.8583

Houston School Sites and Lead Teachers
Jefferson Davis H.S., Lois Range
Lee H.S., Thomas Ivy
Jack Yates H.S., Diane Schranck

California Coordination Center
Tom Hinojosa, *Center Coordinator*
Santa Clara, Calif., 408.244.3080

California School Sites and Lead Teachers
Sherman Indian H.S., Mary Yarger
Sacramento H.S., Brian Jacobs

Iowa Coordination Center
Robert Yager, *Center Director*
University of Iowa, 319.335.1189

Iowa School Sites and Lead Teachers
Pleasant Valley H.S., William Roberts
North Scott H.S., Mike Brown

North Carolina Coordination Center
Charles Coble, *Center Co-Director*
Jessie Jones, *School Coordinator*
East Carolina University, 919.328.6172

North Carolina School Sites and Lead Teachers
Tarboro H.S., Ernestine Smith
Northside H.S., Glenda Burrus

Puerto Rico Coordination Center*
Manuel Gomez, *Center Co-Director*
Acenet Bernacet, *Center Co-Director*
University of Puerto Rico, 809.765.5170

Puerto Rico School Site
UPR Lab H.S.

* * * * * * * * * * * *

Pilot Sites
Site Coordinator and Lead Teacher
Fox Lane H.S., New York, Arthur Eisenkraft
Georgetown Day School, Washington, D.C., William George
Flathead H.S., Montana, Gary Freebury
Clinton H.S., New York, John Laffan*

*not part of the NSF-funded SS&C Project.

Project Associates

Bill G. Aldridge
SciEdSol, Henderson, Nev.

Dorothy L. Gabel
Indiana University

Stephen G. Druger
Northwestern University

George Miller
University of California-Irvine

National Science Education Standard—Physical Science

Motions and Forces

a. Between any two charged particles, electric force is vastly greater than the gravitational force. Most observable forces such as those exerted by a coiled spring or friction may be traced to electric forces acting between atoms and molecules.

Interactions of Energy and Matter

b. Waves, including sound and seismic waves, waves on water, and light waves, have energy and can transfer energy when they interact with matter.

Teacher Materials

Learning Sequence Item:

1018

Hooke's Law, Vibrations, Mechanical Waves, and Sound

February 1997
Adapted by: Bill G. Aldridge

a. Elastic and Frictional Forces: Electric Forces Between Atoms and Molecules. Using a graph of force vs. extension of a spring students should be able to find the slope and relate it to the spring constant k, observing that different slopes represent different springs of different stiffness *(Physics, A Framework for High School Science Education, p. 24).*

b. The Wave Model: Water Waves, Seismic Waves, Sound, and Light. Students should develop the concept of standing waves and observe the resonance of sound waves caused by vibrating strings and air columns *(Physics, A Framework for High School Science Education, p. 38).*

Contents

This micro-unit was adapted by **Bill G. Aldridge** (SciEdSol, Henderson, Nev.)

1018
Learning Sequence

a. Elastic and Frictional Forces: Electric Forces Between Atoms and Molecules. Using a graph of force vs. extension of a spring students should be able to find the slope and relate it to the spring constant k, observing that different slopes represent different springs of different stiffness *(Physics, A Framework for High School Science Education, p. 24).*

b. The Wave Model: Water Waves, Seismic Waves, Sound, and Light. Students should develop the concept of standing waves and observe the resonance of sound waves caused by vibrating strings and air columns *(Physics, A Framework for High School Science Education, p. 38.*

Science as Inquiry	Science and Technology	Science in Personal and Social Perspectives	History and Nature of Science
How Do Springy Things Vibrate? **Activity 1** How Do Strings on Musical Instruments Vibrate? **Activity 2** Using a Weak Sound to Make a Loud Sound **Activity 3**	Resonance of a Suspension Bridge **Assessment 1** The Bird on the Wire **Assessment 2**	What Guitars Do **Reading 2** The Quality of Music from a Guitar **Assessment 3**	Explaining the Power of Springing Bodies *(Original observations by Robert Hooke)* **Reading 1**

Suggested Sequence of Events

Event #1

Lab Activity
1. How Do Springy Things Vibrate?

Event #2

Reading 1
Explaining the Power of Springing Bodies *(Original observations by Robert Hooke)*

Reading 2
What Guitars Do

Event #3

Lab Activity
2. How Do Strings on Musical Instruments Vibrate?

Event #4

Lab Activity
3. Using a Weak Sound to Make a Loud Sound

Assessment items are at the back of this volume.

Readings are included with the student version of the unit.

Assessment Recommendations

This teacher materials packet contains a few items suggested for classroom assessment. Often, three types of items are included. Some have been tested and reviewed, but not all.

1. Multiple-choice questions accompanied by short essays, called justification, that allow teachers to find out if students really understand their selections on the multiple choice.

2. Open-ended questions asking for essay responses.

3. Suggestions for performance tasks, usually including laboratory work, questions to be answered, data to be graphed and processed, and inferences to be made. Some tasks include proposals for student design of such tasks. These may sometimes closely resemble a good laboratory task, since the best types of laboratories are assessing student skills and performance at all times. Special assessment tasks will not be needed if measures such as questions, tabulations, graphs, calculations, etc., are incorporated into regular lab activities.

Teachers are encouraged to make changes in these items to suit their own classroom situations and to develop further items of their own, hopefully finding inspiration in the models we have provided. We hope you may consider adding your best items to our pool. We also will be very pleased to hear of proposed revisions to our items when you think they are needed.

Teacher Sheet

Science as Inquiry

How Do Springy Things Vibrate?

Why do some things vibrate faster than others?

Overview:

When your students carried out activities in Micro-unit 921, they saw that when they push or pull on something that is springy, like a rubber band or a spring, it pulls back. They also saw that when they graph the force in newtons needed to extend a spring by a certain amount, with the extension of the spring in meters, the slope of the graph is a measure of the stiffness of the spring. Since a graph with this force on the vertical axis and spring extension on the horizontal axis gives a straight line that goes through the origin (zero for both force and extension), one can describe the relationship with a very simple equation, $F=-KX$, where F is the force on the spring, X is its extension, and the value of the slope of the graph, that we called K, is the spring constant. The minus sign indicates that the spring pulls in a direction opposite to its extension. Students saw that the spring constant K was larger when springs were stiffer. They will now see how a mass attached to a spring vibrates.

Materials:

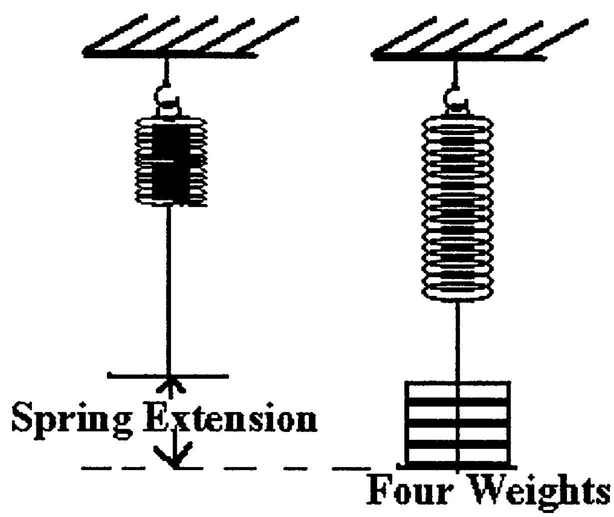
Per class:
set of four springs, each with a different spring constant

Per lab group:
ring stand with supports to hold spring
weights (50 g to 100 g), 5–6, with holders
stop watch
meter stick
notebook and graph paper

Procedure:

Students should attach one end of a spring to a tabletop or ring stand and place some weights onto the other end as shown in the figure. They then pull down on the weights, extending the spring 3 to 4 centimeters. When they let go, the weights will vibrate up and down.

Have them use a stop watch to measure the period of oscillation of the weights. They should count 10 oscillations. This should give a good measure of the period for one oscillation. They should do this for several different initial amplitudes and for at least four different mass values used as weights.



Although students are told to repeat the procedure for at least four different spring constants, it is very repetitive. This is an excellent opportunity to utilize cooperative learning. Assign the four different springs to four different teams, and the teams can then share data at the end of the lab period. Analysis of data should not be done during the lab period, since students will need to aggregate data from other students. This could be done as homework or as an in-class activity.

Background:

The period of oscillation of a mass on a spring, assuming this mass is much greater than that of the spring and that the spring is linear, is given by

$$T = 2\pi(M/K)^{1/2}.$$

This experiment is intended to lead students to this relationship as an empirical law. It can be deduced from fundamentals as a consequence of Newton's second law and certain kinematics laws.

When Newton's laws are applied to a mass on a spring, the resulting equation is a differential equation. There is, however, a situation analogous to the pendulum bob moving in a circle that can be applied to the mass on a spring. But this derivation requires a bit more imagination. Suppose that some small mass is moving in a circular path and that the radius of its path is the same as the amplitude, *A*, of the oscillation of the mass on the spring. Suppose further that the mass moving in the circular path has the same value as that on the spring and that it is moving at exactly the right speed so that when the mass on the spring reaches its topmost position, the mass traveling in a circle is at some point we call *P*. Then suppose that when the mass on the spring reaches its lowest position, the mass moving in the circle is exactly opposite the point *P*, a diameter *2A* away from *P* at point *Q*. Under these conditions, the component of the acceleration along the diameter for the mass moving in the circle is identical with the acceleration of the mass on the spring. Since we know that the acceleration of the mass moving in the circle has a magnitude of v^2/A, and that the speed v is given by distance $2\pi A$ divided by time *T*, we have for this acceleration

$$a = (2\pi A/T)^2/A,$$

or more simply $4\pi^2 A/T^2$. Using the fact that $F=Ma$, the force that the spring must exert at the analog to point Q must be

$$F = 4\pi^2 MA/T^2.$$

But the spring would be extended from its equilibrium position by an amount $X = A$ at this point. Therefore, the spring must exert a force of $F=KA$. Setting these two forces equal to each other, we have

$$4\pi^2 MA/T^2 = KA.$$

Solving this equation for *T*, we have

$$T = 2\pi(M/K)^{1/2}.$$

When students carried out the experiments involving *M* and *K*, they arrived at the same relationship, but with a constant of 6.28. We can now see that the theory shows that this constant is really 2π.

The importance of this equation cannot be overstated. When it is solved for *K*, we have

$$K = 4\pi^2 M/T^2,$$

or, in terms of frequency *f* since $f=1/T$,

$$K = 4\pi^2 M f^2.$$

Then, if we can measure the frequency of oscillation of a diatomic molecule (for example, by observing which frequencies of light are absorbed by that molecule) and knowing the reduced mass of the molecule ($M/2$ for two atoms of equal mass *M*), we can find the constant *K*, which is a measure of the bond strength. Turning this sequence around, if we know the bond strength, we can estimate the absorption frequency for the molecule. This process is carried out regularly in chemistry, and the infrared spectra of molecules is directly connected to the use of equations like this one. It is not obvious that the frequency of oscillation of the diatomic molecule would necessarily be the same as the photon frequency needed to induce that oscillation. To show that this is true, one must use the fact that energy is quantized as $E=(l+1/2)hf_{osc}$, where l = 0,1,2, . . . To conserve angular momentum, a selection rule is required of the photon absorption. The value of *l* can change only by 1. By conservation of energy,

$$E_{ph} = \text{change in } E = hf_{osc}.$$

As an example, the absorption frequency for a single bond of two carbon atoms is 3.6 x 10^{13} s^{-1}. This frequency corresponds to a wavelength of 8,300 nm, well into the infrared region. The reduced mass of two carbon atoms is six atomic mass units. From this one can compute the constant *K*. It is 516 N/m. Students at this level will not be expected to understand this theory, but the teacher certainly should understand it.

This laboratory activity is an excellent opportunity for students to learn how to control variables. Since the period *T* could conceivably depend upon mass *M*, amplitude *A*, and spring constant *K*, there are three independent variables, *M*, *A*, and *K*, with the dependent variable *T*. Two of the three independent variables must be held constant in each of the three experiments to determine the three relationships, *T = function (M)*, *T = function (A)*, and *T = function (K)*. Only two of these experiments show relationships, those for *M* and *K*.

You may need to help students analyze data to arrive at the square root relationship. If you have advanced students, they might learn how to use log-log graph paper, where the square root shows up as a measured slope of 1/2.

Answers to Student Questions

1. Does the period of oscillation depend upon the amplitude?

Students will observe that the period does not depend upon amplitude. This result is analogous to that for the period of a pendulum, which is also independent of amplitude so long as the amplitude is not too large.

2. For each spring, make a graph with period of oscillation on the vertical axis and total mass attached to the spring on the horizontal axis.

Students will observe a direct relationship of some kind between period and mass, but the graph is not linear; therefore, it is not a simple relationship. Having different spring constants, the graphs for the different springs will look alike but will not have the same initial slope. The stiffer the spring, the smaller the initial slope.

3. If the graph is not a straight line, try graphing the period versus the square or the period versus the square root of the mass. Does either of these approaches give a straight line? If so, what does this mean?

A graph of the period versus the square of the mass will not give a straight line, but a graph of the period versus the square root of the mass does give a straight line. This means that the relationship is a square root relationship, with the period equal to a constant times the square root of the mass. The constant has a value given by the slope of this graph.

4. You have been given the spring constants (stiffness measures for springs). Graph the period of oscillation versus the spring constant and find this relationship as you did for mass in question 3. What is the result?

This is somewhat more difficult. When period is plotted against spring constant, the graph is some sort of inverse relationship. Taking the square root of the spring constant still gives an inverse relationship, but it forms a nice hyperbola, suggesting an inverse proportion. If the student plots period versus the reciprocal of the square root of the spring constant, the result is a straight line.

5. Describe in words the relationship between the period of oscillation of a mass on a spring and the value of the mass. Do the same for the relationship between the period and the spring stiffness. Write these relationships as proportions.

T = constant × square root of M

T = other constant × square root of $(1/K)$, or $T \times K$ = other constant.

The period is directly proportional to the square root of the mass and inversely proportional to the square root of the spring constant.

6. Write the relationships between the frequency of oscillation and the mass and stiffness constants ($f = 1/T$).

f = constant × square root of spring constant

f = other constant × square root of reciprocal of mass

Note: These relationships intuitively suggest the equation, f = constant × square root of (K/M), but this step cannot be done algebraically. It requires the solution of a differential equation. It is a common error of teachers to try to combine such relationships into one (as with the gas laws), but in fact it is impossible to do so algebraically, leading only to circular reasoning.

Adapted from: none

Teacher Sheet

Science as Inquiry

How Do Strings on Musical Instruments Vibrate?

Why does some music sound different than others?

Overview:

Students learned in Activity 1 that the frequency of oscillation of a mass on a spring is inversely proportional to the square root of the mass and directly proportional to the square root of the spring constant (measure of stiffness) of the spring. Although on the surface that result does not appear to be very important, it has an enormous number of applications in science, from the way musical instruments work to the vibrations associated with sound and heat. In this activity students examine a particular example, the vibrations of guitar strings, and the sound that is produced.

Materials:

Per lab group:

guitar
set of the six guitar strings, each 10-cm long
pitch pipe
spring balance, calibrated in newtons
metric balance (microbalance or precise triple-beam balance)

Procedure:

Have students adjust the tension in the thickest guitar string so that it is loose but still under some tension. They should then pull it aside at its midpoint and release it. They will see that it vibrates, much as the mass on the spring vibrated. If the tension is not too great they may hear only a very low pitch or nothing at all. They then tighten the string and repeat the process. Even though they cannot easily see the string vibrating, they can feel it by touching lightly with a finger. Thus they know that guitar strings vibrate to produce sound.

Next students should tune the guitar carefully using the pitch pipe. Try to include one lab partner in each group who knows music and can show how this is done properly.

Students then attach the spring balance to the midpoint of each of the six strings and pull outward until the string has been displaced 1.0 centimeters, recording the force in newtons required to displace each string. These forces are measures of the tension in the string. For each force F, $F = 4xT/L$, where x is the distance the string is pulled sideways at its midpoint, T is the tension in the string, and L is its length from bridge to nut. The tension T would then be given by $T = FL/4x$.

Have students use a microbalance or precise triple-beam balance to measure the masses of each of the lengths of guitar string provided. They should then divide the mass of each string segment expressed in kilograms by its length expressed in meters (0.10 meters).

Make sure that students have their guitars tuned properly. Then have them record the value of force required to displace each string at its midpoint by 1.0 cm. They should calculate the tension in the guitar string for each of these cases using the equation $T= FL/4x$.

Students have been given the open-string frequencies of a properly tuned guitar (in *just intonation,* not even tempered). From thickest to thinnest string, those frequencies are as follows:

Note	**Frequency**
EE	165 Hz
AA	220
D	297
G	396
B	495
E'	660

Have students make a graph with frequency *f* on the vertical axis and mass/length *M/L* on the horizontal axis. If the result is not a straight line (which it is not), they use the methods of Activity 1 to find the relationship between *f* and *M/L*.

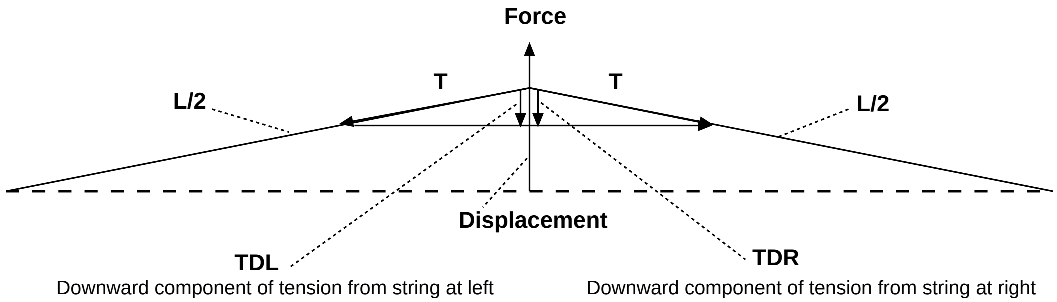
Background:

There are two areas of background interest for the teacher in this activity: how to tune a guitar and how the tension in a string can be measured by its displacement.

Students with experience in music will know how to tune the guitar using a pitch pipe. They may not understand the science that underlies that process, and you should not spend too much time discussing it here. When two pure tones are produced at the same time, you hear a tone having a frequency equal to the sum of the frequencies of the two tones, a tone having a frequency equal to the difference of the frequencies of the two tones, and the frequencies of the two tones themselves. For example, the D string frequency of a tuned guitar is 297 Hz and the G string frequency is 396. If these two strings are strummed at their midpoints simultaneously, you will hear tones corresponding to these two frequencies. But you will also hear a tone corresponding to their sum, 693 Hz, and to their difference, 99 HZ.

If a pitch pipe is sounded at the same time as a string is plucked at its midpoint, you will hear the difference between these two frequencies as you turn the tuning peg. When the string is in tune, the difference will be zero and there will be no "beat frequency" as it is called. Some students have almost a natural ear for tones and can listen to the pitch pipe, remember its sound, and adjust the string to match, doing this in a serial fashion until they are satisfied the string is properly tuned. Since the ear cannot hear beat differences at very low frequencies, an electronic device called an oscilloscope can be used to observe exact points at which the two frequencies are equal.

How do we measure the tension in a guitar string by displacing it at its midpoint? When a string is pulled outward at its midpoint, the force of tension can be determined using similar triangles, as illustrated on the following page.



The triangle formed by the displacement, half the displaced string length *L/2*, and half the original string length has the same angles as the triangle formed by the tension *T*, the downward component of that tension, and the left-directed or right-directed component of that tension. Thus these triangles are similar. Since similar triangles have sides in the same proportions, we can form the equality of ratios as follows:

$$TDL/T = displacement/(L/2) = TDR/T$$

Since force must be equal to $TDL + TDR$ (because the string is in equilibrium and not accelerating, so the net vertical force is zero), and since $TDL = TDR$, we have $force = 2TDR = 2TDL$. Then $x/(L/2) = (force/2)/T$, where $x = displacement$. Simplifying this equation, $force = 4xT/L$ and $T = FL/4x$, where F is force you apply and x is displacement.

Answers to Student Questions:

1. When students pulled each string of a properly tuned guitar out at a center point by the same amount, they could calculate the tension in each string. How did these tensions compare? What did they think would be the design implications for a guitar in terms of stresses on parts of the guitar if the guitar were not made this way?

The tensions in the strings had almost the same values. The thicknesses of the strings designed for guitars are determined so that for a tuned guitar the tensions will be the same for each string. This means that there will be no torques applied to the bridge or the neck of the guitar via the tuning pegs. Such torques could distort or damage the instrument.

2. When students found the relationship between open-string frequency and *M/L*, they found that *f* was inversely proportional to the square root of *M/L*. They are asked to compare this result with that of a mass on a spring.

The relationship is the same in terms of the amount of mass of a unit of length. Thus, if the student divides the mass of a 10-cm length of guitar string by 0.10, finding the mass of 1 meter of such string, this is the value of the quantity *M/L*. The frequency is inversely proportional to the square root of this quantity.

Adapted from: none

Teacher Sheet

Science as Inquiry

Using a Weak Sound to Make a Loud Sound

How does a singer break a glass with her voice?

Overview:

Why does a guitar have a body, bridge, and sound hole? Why do other stringed instruments have similar bodies and holes? How can you produce a sound by blowing across the opening of an empty bottle? Why does a pipe organ have so many long pipes? This activity is designed to answer these questions. Students will have experiences and come to understand the concept of resonance.

Materials:

Per lab group:
guitar
ring stand
weights and weight holder
guitar string
paper, small piece
Slinky®
fine powder (e.g., lycopodium) (optional)

Part 1
Procedure:

To answer the questions raised in the overview, students need to make several observations. First, they should hang a guitar string from a ring stand with a weight attached to its free end. The string should be the same length as the distance between the nut and bridge on the guitar with an attached weight that will give the same force on the string as the tension in the guitar string (use the tension found in Activity 2). This situation results in a string equivalent to the string on the guitar, except there is no bridge, sound board, or sound hole. Have students strum the guitar string and describe what they hear. They should then strum the freely hanging string, which is identical in length and under the same tension as the guitar string, and describe what they hear. They should describe in their own words how the sounds are different.

Next have students select any two adjacent strings on the guitar and strum the one with highest pitch at the midpoint between the nut and the bridge. They then hold the adjacent string down against various frets, strumming it at the midpoint between the bridge and the fret against which it is being held, until they find where it sounds like it has the same pitch as the higher-pitched open string when it is strummed at its midpoint.

When the two strings sound like they have the same pitch, students lightly touch each to make sure they are not vibrating. They then fold and place a small bit of paper so that it hangs at the midpoint of the open string. Holding the adjacent string against the fret to produce the same pitch as the open string and strumming this string at its midpoint, they observe what happens to the bit of paper on the open string. They should then repeat this procedure, but without the bit of paper. After strumming the string, they should quickly touch the center of the string to stop its vibration and describe what they hear from the open string.

Background

What students have observed is a phenomenon given the name *resonance*. Note that they have the experience *before* being given the name for this phenomenon. Also, we have not explained resonance, only named it. There are many other such examples to firm up the concept of resonance in different contexts. If you push a child in a swing and give a little push each time the swing comes back to where you are standing, the swing is resonant with your push. If you rub a crystal glass edge with a damp finger round and round, the glass can be made to "ring." This is resonance. Supposedly it is resonance that causes the glass to break when the opera singer sings. When you stand on a hanging foot bridge and jump up and down with just the right frequency, it begins to oscillate up and down and will ultimately collapse. A famous case of this phenomenon was the Tacoma Narrows Bridge collapse, where wind set up the resonant frequency of the torsional motion of the bridge, leading it to collapse completely. A tragic example of resonance was the collapse of balconies at a Hyatt Regency Hotel in Kansas City, where patrons keeping rhythm to the music at a tea dance were doing so at or near the resonant frequency of the balconies, causing the balconies to collapse and kill and injure many people.

Part 2

Procedure:

To better understand resonance, students can examine the motion of something very simple, the vibrations of a Slinky®. Attaching one end of the Slinky® firmly to a tabletop, students move away from the table until the Slinky® is well-extended. They then hold the free end of the Slinky® and move it up and down quickly. They will observe a pulse moving along the Slinky® that hits the far end and bounces back. They should describe what they observe, noticing that the wave or pulse moves down the Slinky®, hits the fixed end, and returns, but is inverted after reflection.

Now have students start moving the free end of the Slinky® up and down at a very low rate. They will probably see an irregular pattern or behavior of pulses. But as they speed up the rate at which they swing the free end, the wave sent to the fixed end of the Slinky® will reflect and come back just in time to reinforce the wave being sent in the next movement of the Slinky®. This will be the first resonance. They will have formed, with little effort, a standing wave with the center of the Slinky® moving up and down.

As students continue to speed up the rate at which they swing the free end of the Slinky® up and down, they will discover another standing wave, this one at twice the frequency of the first. They will see two antinodes and a node at the midpoint of the Slinky®. With skill, students will be able to produce as many as four or even five antinodes. Thus they will see how standing waves form on strings, even though these cannot be seen the way they can be seen on a Slinky®.

Background:

Even though standing waves cannot be seen on a string, they can be heard. On a guitar string they are called overtones or harmonics. Plucking a guitar string at its midpoint prevents an antinode at that location, so that there can be only harmonics with antinodes at the midpoint. This means there can only be harmonics three times the fundamental, five times the fundamental, etc., with odd multiples of the fundamental frequency. Plucking the string at one-fourth the distance from the bridge would mean an antinode there, allowing a harmonic of two, three, and five times the fundamental, but not one at four times the fundamental. This location would produce several high-frequency harmonics, and those harmonics sound louder than the harmonics produced when a string is plucked at its midpoint.

The richness of the sound of a guitar is therefore produced by plucking strings at some point other than their midpoints, and by having a sound hole and sound board. The string produces many harmonics, and the sound hole and sound board resonate with several of these harmonics and with the fundamental.

Variations:

Students might try various things with a guitar to produce resonances and harmonics. For example, if you sprinkle some fine powder, like lycopodium, on the sound board and then strum one of the guitar strings, what happens to the powder? How can students explain this observation? They might also hold strings down against certain frets after strumming the string and listen to faint sounds of various frequencies. They should offer explanations of these observations in terms of harmonics.

Answers to Student Questions:

1. When a string that is identical to a guitar string and under the same tension is plucked at its midpoint and the sound compared with the sound produced by the same string on a guitar, the differences are quite dramatic. There is no quality to the sound, it is not very loud, and it certainly lacks the quality one hears from a guitar string. This all has to do with resonances set up by the fundamental and its harmonics on the sound board and in the sound hole of the guitar.

2. When a guitar string is plucked, an adjacent open guitar string that is tuned to the same frequency will start to vibrate. It is resonating with the other string, gaining this vibrational energy through the sound board, which couples to the other string. Some of the energy is coupled as sound moves through the air to the other string.

Adapted from: none

Science and Technology

Resonance of a Suspension Bridge

Item:

You walk out to the midpoint of a footbridge suspended over a deep gorge. The footbridge is 100 meters long, and when you are at its midpoint you are able to determine that the gravitational force due to your mass has caused the center of the bridge to be displaced downward 2.0 m. You have a mass of 80 kg, and the value of g at this location is 9.8 m/s^2. Treating the bridge as an approximation of a string, what is the value of the tension in the bridge? If the bridge has a mass of 50 kg/meter, what is its resonant frequency? How many times would you have to jump up and down at the center of this bridge to get it to oscillate at its resonant frequency?

Answer:

The force being applied to produce a displacement of 2.0 meters is 980 newtons, your weight. The tension produced in the bridge by this weight is given by $T = FL/4x$, which, in this case gives a result of 12,250 newtons. So the tension in the bridge is 12.5 times your weight. The fundamental frequency of vibration for this foot bridge is given by the equation $f = (1/6.28)[K/(M/L)]^{1/2}$. In this case, we have $f = .159[(980\ n/2m)/(50\ kg/m)]^{1/2}$. Carrying out these operations, we get a result of $f = 0.5\ Hz$. Thus, this footbridge has a natural frequency of 1/2 oscillation per second. The period of this oscillation is 1/0.5 seconds, or 2 seconds. If you jump up and down so that you land once every two seconds, you will be jumping at the resonant frequency of this footbridge. Before long, its oscillations will be so great as to make it collapse under the enormous tensions.

Science and Technology

The Bird on the Wire

Item:

Suppose that it is winter and the temperature has dropped to –10 degrees Celsius—very cold! A high-voltage transmission wire near your house has become very tight, contracting in length under these cold temperatures. Suppose that a large bird having a mass of 1 kg lands at the midpoint of the transmission line wire, which stretches 300 meters between supporting towers. If the weight of the bird causes the wire to displace downward 1 cm, what tension does the weight of the bird produce in the wire?

Answer:

The bird has a weight given by $w = mg$, or 9.8 newtons. The tension in the wire is given by $T = FL/4x$, so that in this case $T = (9.8\ n)(300\ m)/(0.04\ m) = 73{,}500$ *newtons*. This would be more than 16,000 pounds of tension produced by a bird weighing only 2.2 pounds. The bird would break the wire.

You will notice that high voltage transmission lines hang quite low at their midpoints from one tower to the next. This is done to prevent such a situation from occurring. The wire hangs as a catenary curve (a hyperbolic trigonometric function, like the shape of the arch in St. Louis) until the bird lands. It then assumes a more triangular shape, and in so doing, avoids an increase in tension by any significant degree.

Science in Personal and
Social Perspectives

The Quality of Music from a Guitar

Item:

If you pluck an open AA guitar string one-third of the way from the bridge, what frequencies of sound will you hear in addition to the fundamental of 220 Hz? What frequencies will you not be able to hear that you would hear if you plucked the string at its midpoint?

Answer:

If you pluck the string one-third of the way from the bridge, then there cannot be a node at that position. This means that you cannot hear the third harmonic, 660 Hz, or the sixth harmonic, 1,320 Hz. You will hear the fundamental, 220 Hz, the second harmonic, 440 Hz, the fourth harmonic, 880 Hz, the fifth harmonic, 1100 Hz, the seventh harmonic, 1540 Hz, etc. These frequencies have various amplitudes that decrease in intensity at higher harmonics, with the fundamental being the loudest. This analysis of frequencies is called Fourier analysis. Their resultant sound is a Fourier synthesis.

Consumable Materials

Item	Quantity per lab group	Activity
guitar string	1	3
notebook and graph paper	—	1
paper, small piece	1	3
set of the six guitar strings, each 10-cm long	1	2

Non-Consumable Materials

Item	Quantity per lab group	Activity
guitar	1	2, 3
meter stick	1	1
metric balance (microbalance or precise triple-beam balance)	1	2
ring stand	1	3
ring stand (with supports to hold spring)	1	1
set of four springs, each with a different spring constant	1 per class	1
Slinky®	1	3
spring balance, calibrated in newtons	1	2
stop watch	1	1
weights (50 g–100 g), with holders	5–6	1
weights and weight holder	—	3

Key to activities:
1. How Do Springy Things Vibrate?
2. How Do Strings on Musical Instruments Vibrate?
3. Using a Weak Sound to Make a Loud Sound

References

Student Readings

Hooke, Robert. *Explaining the Power of Springing Bodies*. London, 1678.
Aldridge, B., A. Strassenburg, and G. Waldman, "What Guitars Do." From *The Guitar: A Module on Wave Motion and Sound*, 1972.