

SCOPE, SEQUENCE, and COORDINATION

A National Curriculum Project for High School Science Education

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SS&C Research and Development Center

Gerry Wheeler, *Principal Investigator*
Erma M. Anderson, *Project Director*
Nancy Erwin, *Project Editor*
Rick McGolerick, *Project Coordinator*
Arlington, Va., 703.312.9256

Evaluation Center

Frances Lawrenz, *Center Director*
Doug Huffman, *Associate Director*
Wayne Welch, *Consultant*
University of Minnesota, 612.625.2046

Houston SS&C Materials Development and Coordination Center

Linda W. Crow, *Center Director*
Godrej H. Sethna, *School Coordinator*
University of Houston-Downtown, 713.221.8583

Houston School Sites and Lead Teachers

Jefferson Davis H.S., Lois Range
Lee H.S., Thomas Ivy
Jack Yates H.S., Diane Schranck

California Coordination Center

Tom Hinojosa, *Center Coordinator*
Santa Clara, Calif., 408.244.3080

California School Sites and Lead Teachers

Sherman Indian H.S., Mary Yarger
Sacramento H.S., Brian Jacobs

Iowa Coordination Center

Robert Yager, *Center Director*
University of Iowa, 319.335.1189

Iowa School Sites and Lead Teachers

Pleasant Valley H.S., William Roberts
North Scott H.S., Mike Brown

North Carolina Coordination Center

Charles Coble, *Center Co-Director*
Jessie Jones, *School Coordinator*
East Carolina University, 919.328.6172

North Carolina School Sites and Lead Teachers

Tarboro H.S., Ernestine Smith
Northside H.S., Glenda Burrus

Puerto Rico Coordination Center*

Manuel Gomez, *Center Co-Director*
Acenet Bernacet, *Center Co-Director*
University of Puerto Rico, 809.765.5170

Puerto Rico School Site

UPR Lab H.S.

* * * * *

Pilot Sites

Site Coordinator and Lead Teacher
Fox Lane H.S., New York, Arthur Eisenkraft
Georgetown Day School, Washington, D.C.,
William George
Flathead H.S., Montana, Gary Freebury
Clinton H.S., New York, John Laffan*

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Project Associates

- Bill G. Aldridge**
SciEdSol, Henderson, Nev.
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University of California-Irvine

Student Materials

Learning Sequence Item:

1018

Hooke's Law, Vibrations, Mechanical Waves, and Sound

February 1997

Adapted by: Bill G. Aldridge

Contents

Matrix

Suggested Sequence of Events

Lab Activities

1. How Do Springy Things Vibrate?
2. How Do Strings on Musical Instruments Vibrate?
3. Using a Weak Sound to Make a Loud Sound

Readings

1. Explaining the Power of Springing Bodies
(Original observations by Robert Hooke)
2. What Guitars Do

Science as Inquiry

How Do Springy Things Vibrate?**Why do some things vibrate faster than others?****Overview:**

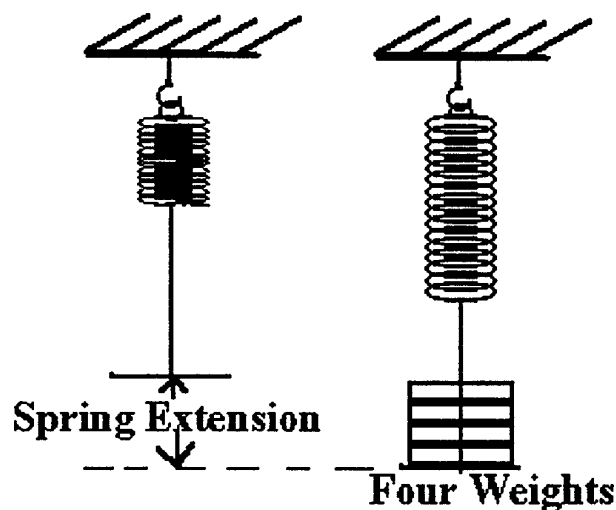
When you carried out activities in Micro-unit 921, you saw that when you push or pull on something that is springy, like a rubber band or a spring, it pulls back on you. You also saw that when you graph the force in newtons needed to extend a spring by a certain amount, with the extension of the spring in meters, the slope of the graph is a measure of the stiffness of the spring. Since a graph with this force on the vertical axis and spring extension on the horizontal axis gives a straight line that goes through the origin (zero for both force and extension), you can describe the relationship with a very simple equation, $F = -KX$, where F is the force on the spring, X is its extension, and the value of the slope of the graph, that we called K , is the spring constant. The minus sign indicates that the spring pulls in a direction opposite to its extension. You saw that the spring constant K was larger when springs were stiffer.

Let us now see how a mass attached to a spring vibrates.

Procedure:

Attach one end of a spring to a tabletop or ring stand and place some weights onto the other end as shown in the figure. Then pull down on the weights, extending the spring about 3 to 4 centimeters. Let go and watch the weights vibrate up and down. Use a stop watch to measure the period of oscillation of the weights. You will recall that the period is how long something takes to go from its present position to some new position and return (from Micro-unit 1015).

Using your stop watch and counting 10 oscillations should give you a good measure of the period for one oscillation. Do this for several different initial amplitudes (defined in 1015) and for at least four different mass values used as weights. Repeat these observations with springs of different stiffnesses.

**Questions:**

1. Does the period of oscillation depend upon the amplitude?
2. For each spring, make a graph with period of oscillation on the vertical axis and total mass attached to the spring on the horizontal axis.

3. If the graph is not a straight line, try graphing the period versus the square or the period versus the square root of the mass. Does either of these approaches give a straight line? If so, what does this mean?
4. You have been given the spring constants (stiffness measures for springs). Graph the period of oscillation versus the spring constant and find this relationship as you did for mass in question 3. What is the result?
5. Describe in words the relationship between the period of oscillation of a mass on a spring and the value of the mass. Do the same for the relationship between the period and the spring stiffness. Write these relationships as proportions.
6. Write the relationships between the frequency of oscillation and the mass and stiffness constants ($f = 1/T$).

Science as Inquiry

How Do Strings on Musical Instruments Vibrate?**Why does some music sound different than others?****Overview:**

You have just learned in Activity 1 that the frequency of oscillation of a mass on a spring is inversely proportional to the square root of the mass and directly proportional to the square root of the spring constant (measure of stiffness) of the spring. Although on the surface that result does not appear to be very important, it has an enormous number of applications in science, from the way musical instruments work to the vibrations associated with sound and heat.

Let us now examine a particular example, the vibrations of guitar strings, and the sound that is produced.

Procedure:

You have been provided a guitar with six strings as well as additional 10-cm lengths of the same six strings. You have also been given a pitch pipe so that you can tune the guitar. You have a spring balance calibrated in newtons and a metric scale for measuring distances.

Adjust the tension in the thickest guitar string so that it is loose but still under some tension. Then pull it aside at its midpoint and release it. You can see that it vibrates, much as the mass on the spring vibrated. If the tension is not too great you may hear only a very low pitch or nothing at all. Now tighten the string and do the same thing. Even though you cannot easily see the string vibrating, you can feel it by touching the string lightly with your finger. Thus you know that guitar strings vibrate to produce sound.

Now tune the guitar carefully using the pitch pipe. Have one of your lab partners who knows music show how this can be done properly.

Next, attach the spring scale to the midpoint of each string and pull outward until the string has been displaced 1.0 centimeters. Record the force in newtons required for each of the six strings to be displaced at the midpoint by 1.0 centimeters. These forces are measures of the tension in each string. (For each force F , $F = 4xT/L$, where x is the distance you pull the string sideways at its midpoint, T is the tension in the string, and L is its length from bridge to nut. The tension T would then be given by $T = FL/4x$.)

Using a microbalance or precise triple-beam balance, measure the mass of each length of guitar string provided. Then divide the mass of each string segment expressed in kilograms by its length expressed in meters (0.10 meters).

Questions:

1. For a properly tuned guitar, record the values of force required to displace each string at its midpoint by 1.0 cm. Calculate the tension in the guitar string for each of these cases ($T = FL/4x$). How do these tensions compare? What do you think would be the design implications for a guitar in terms of stresses on parts of the guitar if the guitar were not made this way?

2. The frequencies of a properly tuned guitar (in *just intonation*, not even tempered) are from thickest to thinnest string as follows:

<u>Note</u>	<u>Frequency</u>
EE	165 Hz
AA	220
D	297
G	396
B	495
E'	660

Make a graph of frequency f on the vertical axis and mass/length M/L on the horizontal axis. If this is not a straight line, use the methods of Activity 1 to find the relationship between f and M/L . How does this result compare with that for a mass on a spring?

Science as Inquiry

Using a Weak Sound to Make a Loud Sound**How does a singer break a glass with her voice?****Overview:**

Why does a guitar have a body, bridge, and sound hole? Why do other stringed instruments have similar bodies and holes? How can you produce a sound by blowing across the opening of an empty bottle? Why does a pipe organ have so many long pipes?

To answer these questions, you need to make several observations.

Part 1**Procedure:**

Hang a guitar string from a ring stand with a weight attached to its free end. The string should be the same length as the distance between the nut and bridge on the guitar with an attached weight that will give the same force on the string as the tension in the guitar string (use tension found in Activity 2). Now you have a guitar string that is the same as the string on the guitar, except there is no bridge, sound board, or sound hole. Strum the guitar string and describe what you hear. Then strum the freely hanging string and describe what you hear. In your own words, how are the sounds different?

Select any two adjacent strings on the guitar and strum the one with highest pitch at the midpoint between the nut and the bridge. Then hold the adjacent string down against various frets and strum it at the midpoint between the bridge and the fret against which you are holding it until you find where it sounds like it has the same pitch as the open string. When these two strings sound like they have the same pitch, lightly touch them to stop the vibrations.

Now fold and place a small bit of paper so that it hangs at the midpoint of the open string. Holding the adjacent string against the fret, strum it at its midpoint to produce the same pitch as the open string had previously when strummed at its midpoint. What happens to the bit of paper? Repeat the process without the attached bit of paper. After strumming the string, almost immediately touch the center of this string to stop its vibration. What do you hear then? And where is this sound coming from?

What you have observed is a phenomenon given the name *resonance*. There are many other such examples. If you push a child in a swing and give a little push each time the swing comes back to where you are standing, the swing is resonant with your push. If you rub a crystal glass edge with a damp finger round and round, the glass can be made to "ring." This is resonance. Supposedly it is resonance that causes the glass to break when the opera singer sings. When you stand on a hanging footbridge and jump up and down with just the right frequency, it begins to oscillate up and down and will ultimately collapse. A famous case of this kind of resonance was the Tacoma Narrows Bridge collapse, where wind set up the resonant frequency of the torsional motion of the bridge, leading it to collapse completely. A tragic example of resonance was the collapse of balconies at a Hyatt Regency Hotel in Kansas City, where patrons keeping rhythm to the music at a tea dance set the balconies into their resonant frequencies, causing the balconies to collapse and kill and injure many people.

To better understand this phenomenon, you can examine something very simple, the vibrations of a Slinky®.

Part 2**Procedure:**

Attach one end of a Slinky® firmly to a tabletop. Holding the free end of the Slinky®, move away from the table until the coil is well-extended. Then move the free end of the Slinky® up and down quickly. What do you see moving along the coil? Describe what you observe. Notice that this wave or pulse moves down the Slinky®, hits the fixed end, and comes back to you. Notice also that the pulse is inverted after reflection.

Now start moving the free end of the Slinky® up and down at a very low rate, adjusting the rate of oscillation until the wave you send to the fixed end reflects back at just the right time to "add" to the vibration of the entire coil. Soon you will have the Slinky® moving so that its midpoint is moving up and down with steadily increasing amplitude. In fact, if you kept moving the free end up and down too strongly, the Slinky® might get such a large amplitude that it would break. When you are making the Slinky® move up and down at its midpoint with this kind of vibration, you have produced what is called a "standing wave."

Now swing the end of the Slinky® up and down at a faster rate until you get a standing wave pattern that has two segments moving up and down on opposite sides of the Slinky® with the midpoint almost stationary. The stationary point is called a *node*, and the places where the amplitude is greatest are called *antinodes*. How does the frequency of the up-and-down motion of your hand holding the Slinky® compare for the two cases you have observed? See if you can move your hand even faster and get three or even four antinodes. These standing waves are produced when your hand is moving with a frequency that is equal to the resonant frequency of the Slinky®, twice that frequency, three times that frequency, etc. These higher frequencies are called *harmonics*.

Go back to the guitar and strum a string at its midpoint. Then move that midpoint down against a fret. What do you hear? Next strum the same string at a point near the sound hole and then hold the string against a fret at its midpoint. What do you hear now? Describe what you think is happening in terms of resonance and harmonics.

You might try various things with a guitar to produce resonances and harmonics. For example, if you sprinkle some fine powder, like lycopodium, on the sound board and then strum one of the guitar strings, what happens to the powder? How can you explain this observation? You might also hold strings down against certain frets after strumming the string and listen to faint sounds of various frequencies. You should offer explanations of these observations in terms of harmonics.

Questions:

1. When a string identical to a guitar string and under the same tension is plucked at its midpoint, how does its sound compare with the sound produced by the same string on a guitar? What could account for these differences?
2. If you pluck a guitar string, what happens to an adjacent open guitar string tuned to the same frequency? How would you explain this observation in terms of energy and the relationship between the strings?

History and Nature of Science

LECTURES

De Potentia Restitutiva,

OR OF

SPRING

Explaining the Power of Springing Bodies.

To which are added some

COLLECTIONS

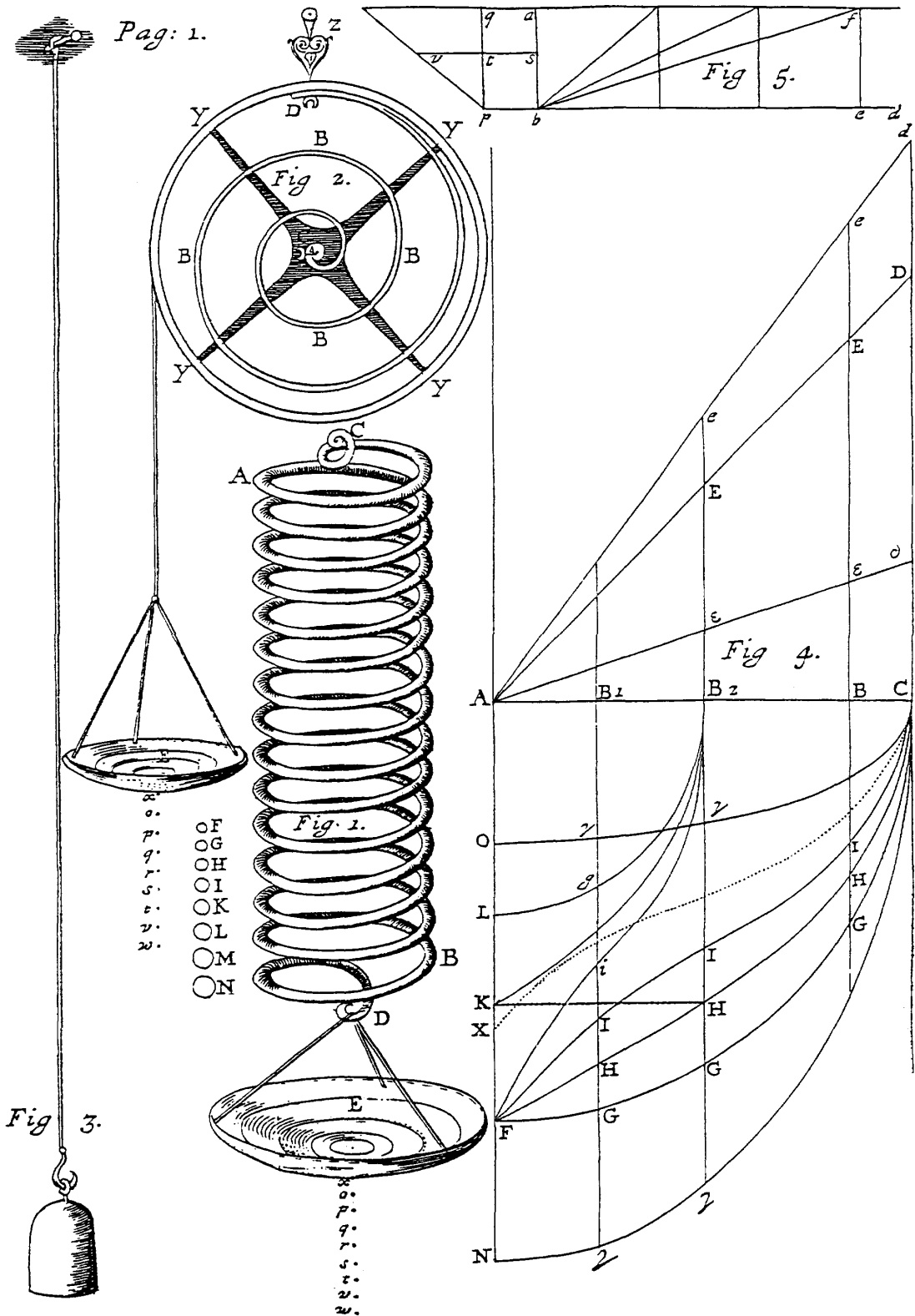
Viz.

*A Description of Dr. Pappins Wind-Fountain and Force-Pump.
Mr. Young's Observation concerning natural Fountains.
Some other Considerations concerning that Subject.
Captain Sturmy's remarks of a Subterraneous Cave and Cistern.
Mr. G. T. Observations made on the Pike of Teneriff, 1674.
Some Reflections and Conjectures occasioned thereupon.
A Relation of a late Eruption in the Isle of Palma.*

By ROBERT HOOKE. S.R.S.

LONDON,

Printed for John Martyn Printer to the Royal Society,
at the Bell in St. Pauls Church-Yard, 1678.



Potentia Restitutiva,

OR

SPRING.



The Theory of Springs, though attempted by divers eminent Mathematicians of this Age has hitherto not been Published by any. It is now about eighteen years since I first found it out, but designing to apply it to some particular use, I omitted the publishing thereof.

About three years since His Majesty was pleased to see the Experiment that made out this Theory tried at *White-Hall*, as also my Spring Watch.

About two years since I printed this Theory in an Anagram at the end of my Book of the Descriptions of Helioscopes, *viz. c e i i i n o s s s t t u u, id est, Ut tensio sic vis*; That is, The Power of any Spring is in the same proportion with the Tension thereof: That is, if one power stretch or bend it one space, two will bend it two, and three will bend it three, and so forward. Now as the Theory is very short, so the way of trying it is very easie.

Take then a quantity of even-drawn Wire, either Steel, Iron, or Brass, and coil it on an even Cylinder into a Helix of what length or number of turns you please, then turn the ends of the Wire into Loops, by one of which suspend this coil upon a nail, and by the other sustain the weight that you would have to extend it, and hanging on several Weights observe exactly to what length each of the weights do extend it beyond the length that its own weight doth stretch it to, and you shall find that if

B one

one ounce, or one pound, or one certain weight doth lengthen it one line, or one inch, or one certain length, then two ounces, two pounds, or two weights will extend it two lines, two inches, or two lengths; and three ounces, pounds, or weights, three lines, inches, or lengths; and so forwards. And this is the Rule or Law of Nature, upon which all manner of Restituent or Springing motion doth proceed, whether it be of Rarefaction, or Extension, or Condensation and Compression.

Or take a Watch Spring, and coil it into a Spiral, so as no part thereof may touch another, then provide a very light wheel of Brass, or the like, and fix it on an arbor that hath two small Pivots of Steel, upon which Pivot turn the edge of the said Wheel very even and smooth, so that a small silk may be coyled upon it; then put this Wheel into a Frame, so that the Wheel may move very freely on its Pivots; fasten the central end of the aforesaid Spring close to the Pivot hole or center of the frame in which the Arbor of the Wheel doth move, and the other end thereof to the Rim of the Wheel, then coyling a fine limber thread of silk upon the edge of the Wheel hang a small light scale at the end thereof fit to receive the weight that shall be put thereinto; then suffering the Wheel to stand in its own position by a little index fastned to the frame, and pointing to the Rim of the Wheel, make a mark with Ink, or the like, on that part of the Rim that the Index pointeth at; then put in a drachm weight into the scale, and suffer the Wheel to settle, and make another mark on the Rim where the Index doth point; then add a drachm more, and let the Wheel settle again, and note with Ink, as before, the place of the Rim pointed at by the Index; then add a third drachm, and do as before, and so a fourth, fifth, sixth, seventh, eighth, &c. suffering the Wheel to settle, and marking the several places pointed at by the Index, then examine the

Distances

Distances of all those marks, and comparing them together you shall find that they will all be equal the one to the other, so that if a drachm doth move the Wheel ten degrees, two drachms will move it twenty, and three thirty, and four forty, and five fifty, and so forwards.

Or take a Wire string of twenty, or thirty, or forty foot long, and fasten the upper part thereof to a nail, and to the other end fasten a Scale to receive the weights: Then with a pair of Compasses take the distance of the bottom of the scale from the ground or floor underneath, and set down the said distance, then put in weights into the said scale in the same manner as in the former trials, and measure the several stretchings of the said string, and set them down. Then compare the several stretchings of the said string, and you will find that they will always bear the same proportions one to the other that the weights do that made them.

The same will be found, if trial be made, with a piece of dry wood that will bend and return, if one end thereof be fixt in a horizontal posture, and to the other end be hanged weights to make it bend downwards.

The manner of trying the same thing upon a body of Air, whether it be for the rarefaction or for the compression thereof I did about fourteen years since publish in my *Micrographia*, and therefore I shall not need to add any further description thereof.

Each of these ways will be more plainly understood by the explanations of the annexed figures.

The first whereof doth represent by A B the coil or helix of Wire, C the end of it, by which it is suspended, D the other end thereof, by which a small Scale E is hanged, into which putting Weights as F G H I K L M N, singly and separately they being in proportion to one another as 1 2 3 4 5 6 7 8, the Spring will be thereby equally stretcht to *o, p, q, r, s, t, u, w,*
B 2
that

that is, if F stretch it so as the bottom of the Scale descend to o , then G will make it descend to p , H to q , I to r , K to s , L to t , M to u , and N to w , &c. So that xo shall be one space, xp , 2, xq , 3, xr , 4, xs , 5, xt , 6, xu , 7, xw , 8.

The second figure represents a Watch Spring coyled in a Spiral by C A B B B D, whose end C is fixed to a pin or Axis immovable, into the end of which the Axis of a small light Wheel is inserted, upon which it moves; the end D is fixed to a pin in the Rim of the Wheel $y y y y$, upon which is coyled a small filk, to the end of which is fixed a Scale to receive the weights. To the frame in which these are contained is fixed the hand or Index z ; then trying with the former weights put into the Scale E, you will find that if F put into the Scale E sinks the bottom of it x to o , then G will sink it to p , and H to q , I to r , K to s , L to t , and z will point at 1, 2, 3, 4, 5, 6, 7, 8 on the Wheel.

The trials with a straight wire, or a straight piece of wood laid Horizontal are so plain they need not an explication by figure, and the way of trying upon Air I have long since explained in my *Micographia* by figures.

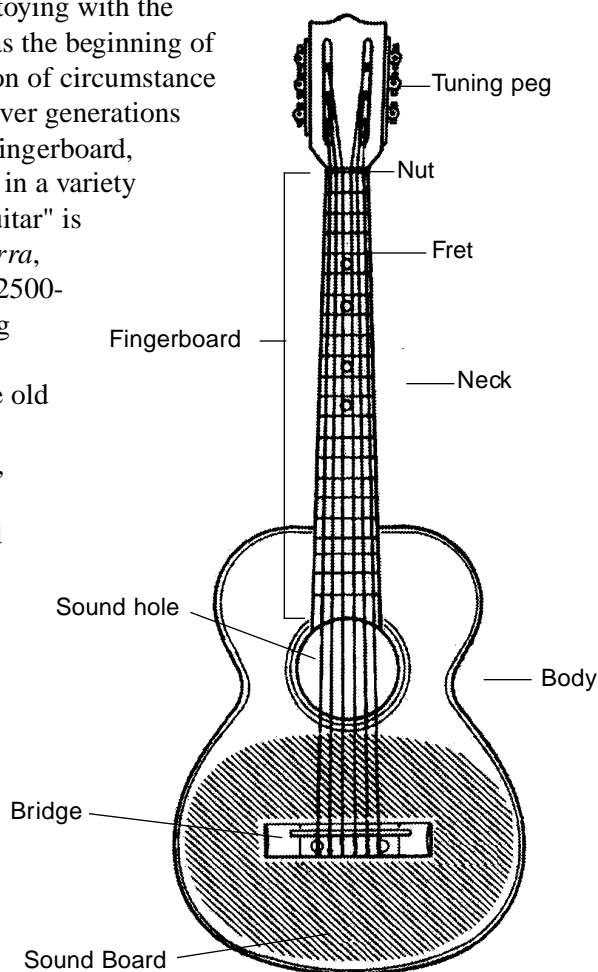
From all which it is very evident that the Rule or Law of Nature in every springing body is, that the force or power thereof to restore it self to its natural position is always proportionate to the Distance or space it is removed therefrom, whether it be by rarefaction, or separation of its parts the one from the other, or by a Condensation, or crowding of those parts nearer together. Nor is it observable in these bodys only, but in all other springy bodies whatsoever, whether Metal, Wood, Stones, baked Earths, Hair, Horns, Silk, Bones, Sinews, Glafs, and the like. Respect being had to the particular figures of the bodies bended, and the advantagious or disadvantageous ways of bending them.

Science in Personal and Social Perspectives

What Guitars Do

Guitars are used to change vibrations of a taut string into sound. Ancient Egyptian and Hittite carvings show guitar-type instruments being made and played as far back as 3000 years ago. One might imagine a man toying with the variations of his bow-string "twang" as the beginning of stringed instruments. By a combination of circumstance and intuition, craftsmen and players over generations have developed the sound board, the fingerboard, more strings, a body, and sound holes in a variety of stringed instruments. Our word "guitar" is much like the old Spanish word *guitarra*, which possibly was derived from the 2500-year-old Sanskrit *chhatur-tar*, meaning "four strings."

The guitar of today is similar to the old Spanish guitar with six strings and an "hour-glass" body. This type of guitar, shown in Figure 1, is one of the most popular of instruments. It is estimated that in the U.S. alone, there are 15 to 20 million guitar owners.



Aldridge, B., A. Strassenburg, and G. Waldman, "What Guitars Do." From *The Guitar: A Module on Wave Motion and Sound*, 1972.