

SCOPE, SEQUENCE, and COORDINATION

A National Curriculum Project for High School Science Education

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National Science Education Standard—Physical Science

Motions and Forces

The rotational motion of ideal rigid objects can be described using angle of rotation in radians, angular speed, and angular acceleration. Methods of use and analysis are mathematical analogs to translational kinematics, so that many of the same methods are applicable.

Interactions of Energy and Matter

Waves, including sound and seismic waves, waves on water, and light waves, have energy and can transfer energy when they interact with matter.

Teacher Materials

Learning Sequence Item:

1015

Measures of Circular Motion

August 1996

Adapted by: Bill G. Aldridge and Paul Mirel

Rotational Kinematics. a) Circular motion should be analyzed to establish the concepts of linear speed, period, and frequency of rotation.

The Wave Model: Water Waves, Seismic Waves, Sound, and Light. b) Students should learn that both a simple pendulum and a mass vibrating on a spring are examples of harmonic motion and that their motion can be described as similar to the motion of a particle in a medium carrying a wave. They should make the connection between periods, amplitudes, and frequencies of harmonic oscillators and the comparable wave descriptors. (*Physics, A Framework for High School Science Education*, pp. 8, 38.)

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2. Chalk It Up
3. Turn, Turn, Turn
4. Change of Motion
5. Wavelengths

1015

Learning Sequence

Rotational Kinematics. a. Circular motion should be analyzed to establish the concepts of linear speed, period, and frequency of rotation.

The Wave Model: Water Waves, Seismic Waves, Sound, and Light. b. Students should learn that both a simple pendulum and a mass vibrating on a spring are examples of harmonic motion and that their motion can be described as similar to the motion of a particle in a medium carrying a wave. They should make the connection between periods, amplitudes, and frequencies of harmonic oscillators and the comparable wave descriptors (*Physics, A Framework for High School Science Education, p. 14*).

Science as Inquiry	Science and Technology	Science in Personal and Social Perspectives	History and Nature of Science
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Suggested Sequence of Events

Event #1

Lab Activity

1. David and Goliath (30 minutes)

Event #2

Lab Activity

2. Bicycle Revolution (40 minutes)

Event #3

Lab Activity

3. Roundabouts (40 minutes)

Event #4

Lab Activity

4. Points of View (40 minutes)

Event #5

Lab Activity

5. Something New? (40 minutes)

Event #6

Readings from Science as Inquiry, Science and Technology, Science in Personal and Social Perspectives, and History and Nature of Science

Suggested readings:

Jewett, J.W., Jr., *Physics Begins with an M . . . Mysteries, Magic, and Myth*. Boston:

Allyn and Bacon, 1994, pp. 106, 114.

Belonuchkin, B.E., "The Fruits of Kepler's Struggle." *Quantum*, Vol. 2, No. 3, Jan./

Feb. 1992, pp. 18–22, 83.

Assessment items are at the back of this volume.

Assessment Recommendations

This teacher materials packet contains a few items suggested for classroom assessment. Often, three types of items are included. Some have been tested and reviewed, but not all.

1. Multiple choice questions accompanied by short essays, called justification, that allow teachers to find out if students really understand their selections on the multiple choice.
2. Open-ended questions asking for essay responses.
3. Suggestions for performance tasks, usually including laboratory work, questions to be answered, data to be graphed and processed, and inferences to be made. Some tasks include proposals for student design of such tasks. These may sometimes closely resemble a good laboratory task, since the best types of laboratories are assessing student skills and performance at all times. Special assessment tasks will not be needed if measures such as questions, tabulations, graphs, calculations, etc., are incorporated into regular lab activities.

Teachers are encouraged to make changes in these items to suit their own classroom situations and to develop further items of their own, hopefully finding inspiration in the models we have provided. We hope you may consider adding your best items to our pool. We also will be very pleased to hear of proposed revisions to our items when you think they are needed.

Science as Inquiry

David and Goliath**What is tangential velocity?****Overview:**

Students construct a simple mass-on-a-string system and use it to optimize the swing of the system for tangential velocity.

Materials:**Per lab group:**

bolt, 4-cm length
meter stick or tape measure
screwdriver
string or twine, 1 m
tennis ball
access to playing field or gymnasium

Procedure:

Students poke a hole in the tennis ball with the screwdriver. They tie one end of the string securely around the middle of the bolt, then push the bolt completely into the ball. The ball should now be firmly affixed to the string.

Using the ball system, students compete to see who can get the ball to go the farthest when it is released—as in the hammer throw in the Olympics. You may wish to make a narrow target corridor to more clearly illustrate the direction of the tangential velocity.

Background:

When released, an object moving in a circular path moves tangentially to the motion—*not* radially. If there were a centrifugal force, it would move radially. There is *only centripetal force*, and when released, the force is zero. So the ball continues its motion in a straight line.

Variations:

None.

Adapted from:

Giancoli, D., *Physics*, 3rd ed., Englewood Cliffs, N.J.: Prentice-Hall Publishers, 1991.

Hewitt, P., *Conceptual Physics*, 7th ed., New York: Harper Collins College Publishers, 1993.

Hewitt, P., *Conceptual Physics High School Program*, 2nd ed., Menlo Park, Calif.: Addison-Wesley Publishers, 1992.

Science as Inquiry

Bicycle Revolution**How can we describe the motion of bicycle wheels?****Overview:**

A bicycle is a nice device that can illustrate the relationships between linear motion and rotational motion. This can be done by comparing the distance and speed that a bicycle moves along a straight line with the motion of the speed and distance traveled by the outer tread of the bicycle wheels. This situation is also an excellent one for considering relative motion.

Materials:**Per lab group:**

bicycle
chalk
tape measure
paper
pen

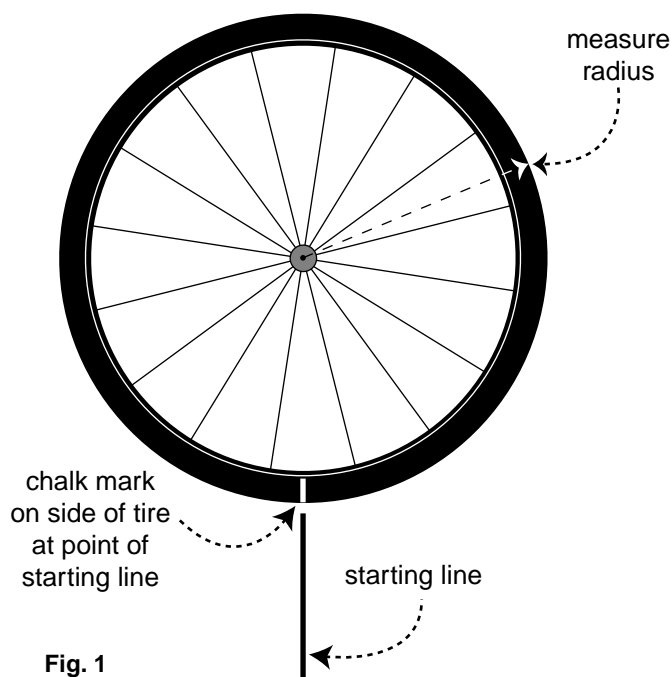
Procedure:

Students measure the distance (in cm) from the center of a bicycle wheel to the outer tread, as shown in Fig. 1. This should be an accurate measure of the radius of that wheel (and tire).

Next, have students place the bike at a starting line, with the front wheel tread just over the starting line. They mark the side of the tire with chalk just above the starting line. Have them walk the bike forward until the front tire has rotated 10 times. The white chalk mark will then be just over the point of contact of the front tire with the floor. They then mark the finish line on the floor at this point. Now the students walk the bike backward, until it is a few feet past the starting line. Have them measure the distance from the starting line to the finish line.

Next, one person rides the bike at a slow constant speed along the same path as used so far. A second person starts the stopwatch at the instant that the chalk mark is over the starting line, then stops the watch at the time the white chalk mark passes just over the finish line.

In this way, students have recorded the time for the bike to travel the distance from the starting line to the finish line, and they will know how long it takes for the bicycle wheel to rotate 10 times, while traveling the linear distance, and at that linear speed.

**Fig. 1**

Background:

This is a basic activity, designed to reveal in a very strong intuitive sense the relationship between linear and rotational motion, and serves as a basis for activities that follow. The complication in this activity arises when one considers the frame of reference from which speeds are determined. Relative to the ground the speed of a spot on the bike tire tread is complicated. But when viewed from the bike's wheel axis (a point through the axle), then the motion is much simpler. The speed relative to the ground varies from 0–2 times that of the bike speed, with an average equal to the bike speed. The speed relative to the wheel axis is always equal to the bike speed.

An interesting observation to make is to have students observe and describe the appearance of a reflector mounted on a bicycle spoke, and viewed at night (using car headlights) while the bike is moving (see “5.” below).

Answers to Student Questions.

1. This should be the measured distance traveled divided by how long the bike took to travel that distance.

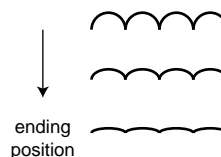
2. Since the bike wheel rotated 10 times and the time of those rotations is how long it took to travel between marks on the floor, the rotation rate must be 10 rotations per time it took. They should divide 10 by the time. They should see that the rotation rate is the same for both wheels, even though they were observing the front wheel.

3. The point on the bicycle tire tread must have traveled the same distance around as the bike moved along the floor. Those distances must be the same. The average speed of this point on the bike tire is the same as the average speed of the bike. Since there are 10 rotations of the bike wheel along the total distance, the distance traveled by the spot on the tread for one rotation is the total distance between marks on the floor divided by 10.

If the bike travels at a constant speed, the spot on the tire tread travels at constant speed equal to that of the bike, but only when taken relative to the bike wheel axis. That axis, however, is moving at the same speed as the bike. Thus, relative to the ground, the spot on the tire tread goes from a maximum speed of twice that of the bike when at its topmost position, to zero at the point when the spot is touching the ground, and, relative to the ground this spot is moving at the same speed as the bike only when the spot is moving at its maximum rate upward or downward.

4. When students divide the distance traveled by the spot on the bike tire tread by the radius of the wheel, they will, of course get an approximation to the quantity 2π . Thus they should get something like 6.28.

5. The reflector would appear as shown in Fig. 2.



As the reflector is moved closer to the axle, its appearance changes as shown.

Fig. 2

Variations:

None.

Adapted from:

None.

Illustrations: M. S. Young

Science as Inquiry

Roundabouts**How can we describe motion along a circular path?****Overview:**

Students previously learned how to describe motion along a circular path, or of the rotation of something like a turntable, using the concept of rpm (revolutions/min, or revolutions/sec). In this activity they will further investigate ways of describing such motion. In particular, how can we find out how long it takes for something revolving or rotating to complete one revolution. What is the relationship between rate of rotation or revolution and the time to rotate or revolve once? How can we determine how fast something is moving when following a circular path? What is the relationship of this velocity and the rate at which the object is rotating or revolving?

Materials:**Per lab group:**

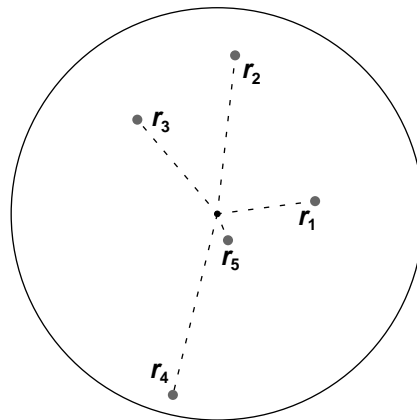
phonograph turntable, $33\frac{1}{3}$ rpm and 45 rpm
felt marker
metric ruler

Procedure:

This investigation could use a variety of different rotating or revolving objects. The simplest would be a point at some location on a phonograph turntable. Some turntables can be set either for 45 rpm, or $33\frac{1}{3}$ rpm, allowing an investigation at two different turning rates.

Using a felt marker, students should put four marks in approximately equal intervals between the center of the turntable and the outer edge. They should place one mark right at the outer edge. This will give five different spots that can be observed in circular motion.

Have students measure the distances from the center of the turntable to each of the spots and record that data. Next, they turn on the turntable and observe the motion of each of the spots. The rotational rate should be at both 45 rpm and $33\frac{1}{3}$ rpm for these observations.

**Background:**

It is very important that students learn to use radian measure. A displacement in radians is independent of the actual distance traveled by something moving in a circular path, and reflects only how many revolutions or parts of a revolution the object has traveled. Similarly, angular velocity measures only how many revolutions or parts of a revolution an object moving in a circular path has traveled per second.

These measures are not restricted to circular motion. Instantaneous motion along any curved path can

be described with these quantities, where the radius of curvature of that path can be determined at that instant.

The following discussion is strictly for teachers. Please do *not* try to discuss material at this level with your class, although an individual, highly motivated student might understand it.

What is useful are the facts that $d/r = \theta$ ($d = r\theta$), $v/r = \omega$, and something not studied in this activity, but nonetheless true, $a/r = \alpha$, and $a = r\alpha$, where a is the acceleration of the spot, r the radius of its motion, and α the angular acceleration in rad/second/second.

These equations allow us to treat rotational motion as an analog to linear motion. We need only replace the linear variable by that variable divided by r , and the resulting angular equation is valid. For example,

$$d = d_0 + vt + (1/2)at^2$$

is the equation for motion, when the starting point is d , the initial velocity is v_0 and the acceleration is a , and motion occurs for time, t . If we divide both sides of this equation by the radius of curvature of motion along a curved path, we get

$$d/r = d_0/r + (v_0/r)t + (1/2)(a/r)t^2$$

From our definitions, we have the correct angular motion equation,

$$\theta = \theta_0 + \omega_0 t + (1/2)\alpha t^2$$

an analog of the linear case. All of the other linear kinematics equations have similarly correct rotational analogs.

Answers to Student Questions.

1. Each spot follows a circular path. Students can measure the radii, and using the fact that the circumference of a circle is $2\pi R$, they can calculate each of the distances traveled in one rotation of a spot. Students will see that there are 2π radii along each circumference, regardless of the spot selected. In three revolutions, there are $3 \times 2\pi = 6\pi$ radii traversed by each spot. In $1/4$ revolution, there are $2\pi/4 = \pi/2$ radii traversed by each spot. (This is, of course, 90° expressed in radians). In $1/9$ revolution, there are $2\pi/6 = \pi/3$ radii traversed by each spot. (This is 60° expressed in radians). The point of this item is that angular displacements are expressed in radians, not usually in degrees. Students can see this on their scientific or engineering calculators, where they must set it for either degrees or radians.

2. To find the speed of a spot, students need the period of revolution. For $33^{1/3}$ rpm, the period must be its reciprocal, or 0.03 minutes, or 1.8 second. At 45 rpm, the period must be $1^{1/3}$ seconds. The speed of a spot is determined by taking its circumference distance divided by this period of time. When they divide this speed by the radius for each spot, the results are the same for all spots. They get the angular velocity in radians/second. At $33^{1/3}$ rpm, the angular velocity is just 3.6π rad/sec (about 11.3 rad/sec), and at 45 rpm, the angular velocity is 2.6666π , (about 8.38 rad/sec). The significance of these results is that angular velocity is independent of the radius of the moving spot. All spots have the same angular velocity when the rotation rate is the same.

3. This is an item designed to help students understand the reciprocal relationship between period and frequency, $fT = 1$. They should see that it can be written, $f = 1/T$ or $T = 1/f$.

4. As pointed out in the answer to Question 2, these speeds are 11.3 rad/sec for $33\frac{1}{3}$ rpm and 8.38 rad/sec for 45 rpm. The equation that relates the angular speed and the ordinary speed of the spot is given by $v/r = \omega$, where v is the magnitude of the tangential velocity of the moving spot, r is the radius of its circular motion, and ω is the angular velocity in radians/second. For angular displacement and distance in a circular path, the equation is $d/r = \theta$, where d is distance traveled by the spot, r is the radius of that circular motion, and θ is the angular displacement in radians.

5. Velocities have both a magnitude and a direction. In these cases we have dealt only with magnitudes. The directions associated with measures of rotational motion are more complicated than with regular kinematics of particles. These aspects of motion along curved paths will be treated at a higher grade level.

Variations:

None.

Adapted from:

None.

Illustration: M. S. Young

Science as Inquiry

Points of View**How does rotational motion compare with the motion of a pendulum?****Overview:**

Students have learned about rotational motion, and they have learned how to describe such motion using terms like frequency, period, and angular speed. In this activity they will make observations to see if there is a connection between rotational motion and the back and forth motion of a pendulum.

Materials:**Per lab group:**

string
steel ballbearing
tape or glue
flood lamp
poster board, white

stopwatch

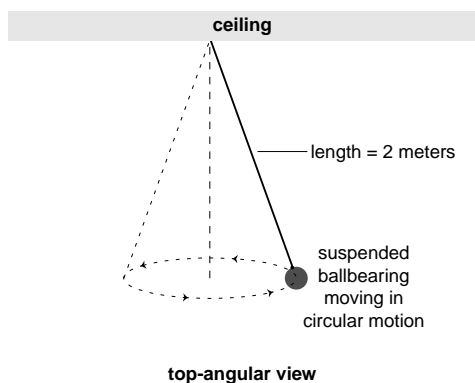


Fig. 1

Looking down onto this motion, students choose a starting point and count 10 revolutions while timing the motion with a stopwatch. From this data they can determine the period and frequency associated with this motion.

Next students kneel or lay on the floor in such a way that they can see just the side view of this circular motion. One student starts the ballbearing moving in a circular motion—as before. Figure 2 shows how this circular motion would appear as viewed from the side.

Students stop the ballbearing and, instead of having it move in

Procedure:

Students will attach a small steel ballbearing to a string having a length of about 2 meters. As shown in Fig. 1, they suspend this pendulum device from the ceiling. Then they throw the steel ball sideways so that it moves nearly in a circular path. This may be hard to do, as the shape of the path followed depends upon how students launch the ball. It most likely will move in an elliptical path. But if they try several times, they should be able to get the motion to look very nearly circular.

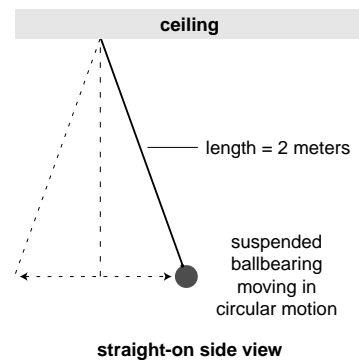


Fig. 2

circular motion, they have the ballbearing move from side to side—as a normal pendulum would move, all the motion being in one plane.

Background:

The fundamental point of this observation is that circular motion has the same measures as oscillatory motion.

The following information is strictly for the teacher. It should *not* be discussed with the 10th Grade class. The equation for the x and y components of the circular motion of an object in the x - y plane are as follows:

$$x = R \cdot \text{Cos}(2\pi t/T)$$

$$y = R \cdot \text{Sin}(2\pi t/T)$$

where R is the radius of the circular path, and x and y are the coordinates of the ballbearing at time t , when the period of revolution is T .

If this motion is viewed in the x - y plane, but along the y -axis, one only sees the x -motion. Therefore the observed motion would be just the equation,

$$x = R \cdot \text{Cos}(2\pi t/T)$$

But the motion of a pendulum with small amplitude $A = R$, and period $T = T$, along that same x -axis would have the equation:

$$x = A \cdot \text{Cos}(2\pi t/T)$$

Since $A = R$, these two equations are identical, and the observed motion is identical.

Now a caveat. The equation for the motion of a pendulum is only approximate. It is true only for small oscillations. Small means that the amplitude is much smaller than the pendulum length. For a pendulum of length 200 cm and amplitude 20 cm, the length is 10 times the amplitude, and this relationship will follow rather closely.

Answers to Student Questions.

1. Students will observe motion very much like a pendulum, but it is really the projection of circular motion. They will find the period of *oscillation* is the same as the period of revolution. The frequency is its reciprocal, $1/T$.

2. Students will observe that the period of a pendulum (for small amplitudes) is the same as the period revolution of the ballbearing moving in a circular path. This observation is important in connecting oscillatory motion with circular motion.

3. The amplitude decreases with time, friction with the air and point of support causing a damping of the oscillation. In the case of

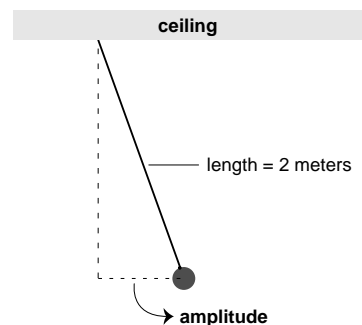


Fig. 3

the ballbearing moving in a circular path, the amplitude is the radius of the circular path, and that radius does decrease with time, as friction slows the motion (Fig. 3). This can be discussed in terms of satellites around Earth, and the air resistance they experience, slowing them down, and reducing their altitude (orbital radius).

Variations:

None.

Adapted from:

None.

Illustrations: M. S. Young

Science as Inquiry

Something New?**How can the motion of a pendulum be recorded? What does the record mean?****Overview:**

Your students have learned that the motion of a pendulum can be described using some of the same terms as the motion of an object moving in a circular path. The period of oscillation of a pendulum corresponds to the period of revolution of an object in a circular path. The amplitude of the pendulum oscillation corresponds to the radius of the circular path. Indeed, when your students view an object moving in a circular path in just the right way, they do not see it moving in a circle. They see it moving back and forth in exactly the same way as a pendulum bob would move with an amplitude the same as the radius of the circular path. Now, what other kinds of motions are there in nature that have some of these same kinds of properties?

First your students will observe something; then they will be asked to describe other kinds of motions that fit what they have observed.

Materials:**Per lab group:**

- stopwatch
- very strong string (or fishing line), 3 meters
- ballpoint pen
- bowling ball, 6–10 lb
- screw with hook end
- butcher paper, 3–5 meters long
- meter stick

Procedure:

Students make a pendulum and bob using very strong string and a bowling ball. (You can get new bowling balls for about \$40 and used ones much cheaper. The mass of bob must be large enough that the friction due to the brush against the paper will not cause a deviation of the pendulum path, or provide excessive damping of oscillations.)

The pendulum must be long enough to almost reach the floor from the ceiling—but leave about 3 cm between the floor and the ball. This pendulum will have a period between 2.5 s and 3.0 s. Have a hole drilled in the ball (“bottom”) in which a pen can be placed so that the ball’s weight presses the pen against the paper. However, the hole also needs to be deep enough for the pen to move freely up and down by 2–3 cm. You must also attach a screw (with a hooked-end) to the bowling ball opposite from the location of the drilled-hole (pen holder).

Students unroll some butcher paper (also called Kraft paper) until they have a piece about 4 meters long (about 90 cm wide). They place the pen in the drilled-hole of the bowling ball so that when the ball is hanging, the pen touches the floor. They need to attach the ball to the ceiling (using the string and

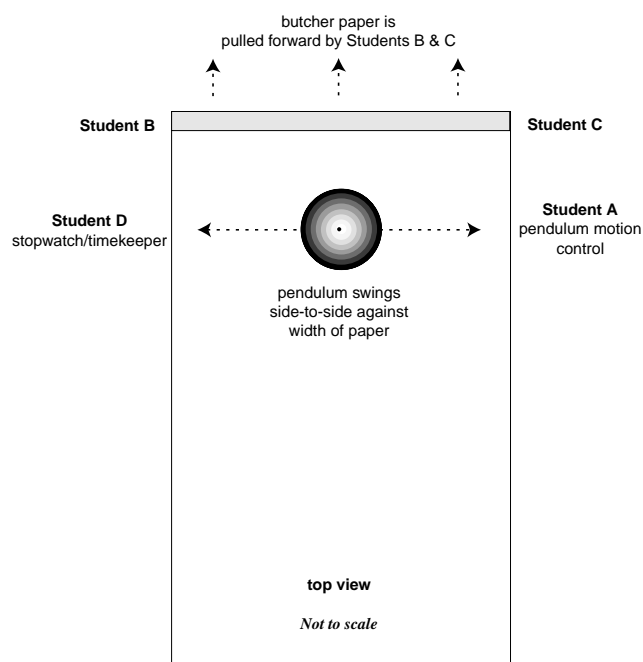
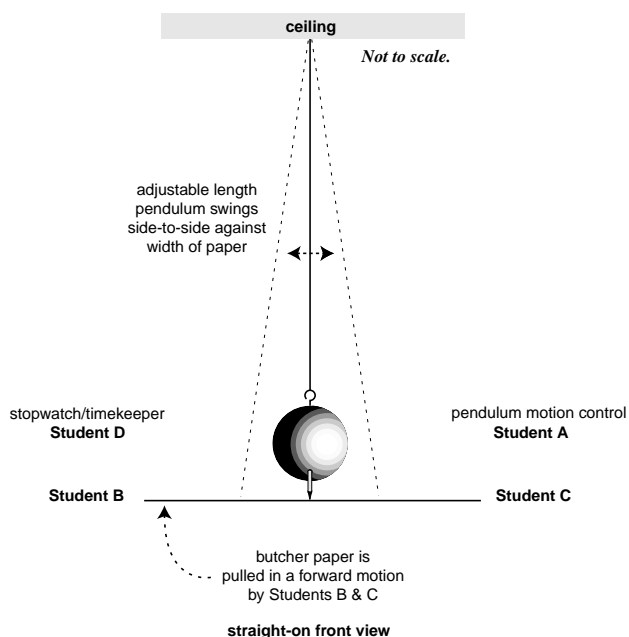
screw) in such a way that they can adjust the length as needed. The entire pendulum setup must make a mark of 5–10 cm across the paper as it swings while the paper is pulled beneath it.

For the observations, students place the butcher paper underneath the stationary pendulum, with the center of the paper under the bowling ball.

Student A pulls the pendulum bob (bowling ball) out about 10–15 cm sideways—the pen must still touch the paper—and holds it stationary. Students B and C prepare to pull the paper from beneath the pendulum. Student D holds a stopwatch to measure the time involved. *All of this must be timed correctly.*

When everyone is ready, Student D (with the stopwatch) does a countdown. At a count of 3, Students B and C pull the paper along the floor at a constant speed, while at the same time, Student A releases the pendulum—so that it swings side-to-side, with the pen marking the paper as it swings back and forth. Student A also counts the pendulum's oscillations over and back, so that its period can later be calculated, while Student D (stopwatch) measures the time.

Just before there is almost no paper left under the pendulum bob, Student D (stopwatch) says “stop.” All at once, Students B and C stop pulling the paper, Student A stops the pendulum, and Student D stops the clock. As students quickly discover, none of this will be done very fast. Since the pendulum has a period of about 2.5–3.0 seconds (s), and we want about 4–6 cycles on the paper, for the length of paper we are using, it must be pulled slowly. For 8 cycles along 2 meters (m) of the paper (25 cm wavelength), with a 2.5 s period, students would need to pull the paper at a speed of $2 \text{ m}/(8 \times 2.5 \text{ s}) = 0.10 \text{ m/s}$, or about 10 cm/s. Students then have a painted record of the pendulum's motion, the time of the motion, and the number of swings of the pendulum.



Background:

There is a fundamental relationship between the oscillation of a pendulum, or of the vibration of something, and the sinusoidal wave pattern. Waves describe vibrations, real waves induce vibrations, and vibrations produce waves. Since vibrations are related to circular motion, the functions that describe waves and vibrations are called circular functions. Circular functions are the trigonometric functions, sine, cosine, etc. We usually use either the sine or cosine to describe waves. The wave produced from an initial amplitude, as in this case where the bob was held out some distance, A_0 , is a cosine wave. (The cosine of 0 has a value of 1). Thus the graph could be described by:

$$A = A_0 \cdot \text{Cos}(2\pi t/T),$$

where A is the amplitude of the wave at some time t , T is the wave period, and A_0 is the maximum amplitude. We have only used the term amplitude for maximum amplitude, but, this is a more general statement of its meaning.

Students should be carefully led to the conclusion that $\lambda f = v$. This needs to be done in two steps. First, they must fully understand the reciprocal relationship of frequency f and period T . To do this, they must see several concrete examples. Thus, give them such cases, e.g., $T = 1$ s, what is the frequency? $T = 2$ s, what is the frequency? $T = (1/2)$ s, what is the frequency? $T = (1/10)$ s, what is the frequency? Then reverse and say, $f = 1$ Hz, what is the period? $f = 1/2$ Hz, what is the period? etc. Students should then use these concrete examples to understand that they can be generalized into the equation $fT=1$, and that it could appear as $f = 1/T$ or $T = 1/f$.

Once students understand this important reciprocal relationship, $fT=1$, then they can be led to understand the relationship, $\lambda f = v$. Use several concrete examples of distance over time equals speed. This is simple kinematics that they understand. Thus, use $6 \text{ cm}/2 \text{ s} = 3 \text{ cm/s}$, etc. Then suggest that these distances are wavelengths, and the times are periods. So use concrete numbers for waves, e.g., 6 cm wavelength, 2 second period, to get a wave speed of $6 \text{ cm}/2 \text{ s} = 3 \text{ cm/s}$. With several examples, they should be able to arrive at the generalization, $\lambda/T = v$.

Once students have the generalization $T = 1/f$ and the generalization, $\lambda/T = v$, they can be guided to the idea of writing $\lambda/T = v$ in the form, $\lambda(1/T) = v$, from which they see that this is the same as $\lambda f = v$. This basic wave relationship has extraordinary application in numerous real world situations, so it is very important.

Answers to Student Questions.

1. They will see a sinusoidal wave pattern on the butcher paper. If they pulled the paper at nearly a constant speed, and the paper was pulled at right angles to the motion of the pendulum, there will be several cycles painted on the paper. They should recognize this pattern as a wave. They should also be able to describe water waves as examples, and perhaps recognize that sound also behaves this way, as well as earthquakes. If they have made waves on wave tanks or slinkys, they may also cite these examples.

2. The distance from one extreme position to the other is twice the amplitude. So students should be able to ascertain that the amplitude is the distance from the rest position of the pendulum bob (center of paper pattern) to extreme value. Their sketch of the basis pattern should be a single sinusoidal wave, of one wavelength. The average length of the repeating wave patterns, is, of course, the wavelength for the waves on the paper.

3. Students measured the time to pull the paper from the starting marks on the paper to the last mark

on the paper. During this same time, they counted the number of oscillations of the pendulum. Dividing the time required by the number of complete oscillations of the pendulum, they can determine the pendulum's period. They should then be able to associate this period with the time for the basic wavelength pattern on the paper to complete one cycle. They should then recognize that the period of the wave is the same as the period of the pendulum. They have learned that frequency and period are reciprocals of each other. Thus, they know that $1/T$ is the frequency. They should be able to express this frequency in oscillations or cycles per second. You should inform them that this unit is given the name Hertz, and abbreviated Hz. Finally, they should recognize that the wave on the paper has a frequency equal to the frequency of the pendulum.

4. This speed calculation not only gives the speed of the paper; it gives the wave speed. When students divide the wavelength (average length of a repeating pattern) by the period, they get the wave speed. When they multiply the wavelength by the frequency, they also get the wave speed. This result is very important, and you need to help them generalize it. Use many examples for them to go from concrete examples to the generalization, $\lambda f = v$. They should also see that this is the same as distance over time equals speed, as expressed in $\lambda/T = v$.

5. The amplitude is half the distance from trough to crest for the wave. Thus the amplitude is 45 cm. The period is just 2 seconds, the time between wave crests. The frequency of these waves is $1/T$, or 0.5 Hz. You might ask students what additional information they would need to get the wavelength of these waves or the wave speed?

6. Here is an application of the relationship, $\lambda f = v$. In this case they know the wave speed and they know the frequency. Thus they can write the equation,

$$335 \text{ m/s} = (440 \text{ 1/s})\lambda$$

where the unit Hz has been replaced by 1/s. They need to understand that 1/s means one complete cycle or oscillation over seconds. Then, when they solve this equation for the wavelength, they get:

$$\lambda = (335 \text{ m/s})/(440 \text{ 1/s}) = 0.76 \text{ meters}$$

or about 76 cm.

Variations:

None.

Adapted from:

None.

Illustrations: M. S. Young

Science as Inquiry

Far and Away**Item:**

How far does a spot on the tread of a bicycle tire travel on a 25-mile bike ride? How many times does a bicycle wheel rotate on that bike ride?

Answer:

The spot on the tread must travel the same distance around and around as the bike did along the path. It must travel 25 miles.

The circumference of the bike tire tread is found by taking 2π times the radius of the tire. (This is the distance from the center of the wheel to the tire tread that touches the ground.) If the distance of 25 miles, expressed in the same units as are used to measure the tire radius, is divided by the tire circumference, you have the number of revolutions of the wheel in 25 miles.

Science as Inquiry

Chalk It Up**Item:**

Suppose that we put a chalk mark on the bottom of a bicycle tire. Then where is the chalk mark when its speed relative to the ground is twice that of the bike? Where is the chalk mark when its speed relative to the ground is 0?

Answer:

The spot on the tread is traveling at all times at a speed equal to that of the bike, but this is relative to a frame of reference at the wheels axis. That frame of reference is moving with the bike and traveling at the bike's speed. Thus, when the spot on the tread is at its topmost position, it is traveling at a speed by its speed relative to the bike wheel axis plus the speed of that axis relative to the ground. Thus, relative to the ground, the spot is traveling at twice the speed of the bike.

At its lowest position, when the spot touches the ground, it is obvious the speed relative to the ground is 0. Otherwise the bike would be skidding. But at this point the spot is still moving at the bike's speed relative to the wheel axis. It is just that the wheel axis is going one way at the bike's speed, and the spot is moving in an opposite direction at the bike's speed, and the sum is 0.

Science as Inquiry

Turn, Turn, Turn**Item:**

An electric motor turns at 1,200 rpm. What is the period of revolution of the motor? What is the angular velocity of this motor? If the motor is connected to a grinder having a radius r , through how many radii does this motor turn during two minutes?

Answer:

If the motor turns at 1,200 rpm, then it must turn at $1,200/60$ revolutions per second. This would be 20 rev/s. This is its rotational frequency, f . Since $fT = 1$, we must have $T = 1/f$, and $T = 1/20$ s, or $T = 0.05$ s. Since angular velocity is the number of radians per second, we must use the fact that there are 2π radians in one revolution, so that the motor must be turning at 40π rad/s, or about 125.7 radians/s. In two minutes, or 120 seconds, the radius r included, will turn $120 \text{ sec} \times 125.7 \text{ radians/s} = 15,084$ radians. Thus, at any distance from the center of the rotation outward, the motor will turn 15,084 radii in 120 seconds.

Science and Technology

Change of Motion**Item:**

You have investigated a situation where you saw circular motion as equivalent to the back and forth motion of a pendulum. Since you could go from circular motion to oscillatory motion, can you go the other way? Can you design something that will allow you to produce circular motion from something that is moving back and forth along a straight line? Can you think of something practical where circular motion is converted to straight line motion?

Answer:

This answer is, of course, central to the design of reciprocating engines. A piston goes up and down along a straight line. It is connected to a crankshaft where that straight line motion is converted to circular motion. When a motor is used to drive some kinds of compressors, the compressor has a piston, much like a reciprocating motor, and we convert circular motion of an electric motor to back and forth motion of a piston in the compressor.

Science as Inquiry

Wavelengths**Item:**

A magnitude 6.8 earthquake in Borneo on March 27, 1969, was observed in Berkeley, Calif. The Rayleigh surface waves produced 8 complete vertical vibrations of the seismograph in three minutes. If these waves traveled at 2.7 Km/s, what was their wavelength?

Answer:

The period of the waves was $180 \text{ s}/8 = 22.5$ seconds. Using the relationship:

$$\lambda f = v$$

we have:

$$\lambda = v/f = vT = (2.7 \text{ Km/s})(22.5 \text{ s}) = 60.75 \text{ km.}$$

A crest of this very wave train passes a point at Berkeley every 22.5 seconds. It goes from crest to trough in 11.25 seconds. So the Earth's surface at Berkeley would move up and down once each 22.5 seconds.

Consumable Materials

Item	Quantity per lab group	Activity
ball bearing, steel	1	4
butcher paper	1 sheet, 3–5 meters	5
chalk	1 piece	2
felt marker or ballpoint	1	3
floodlamp	1	4
paper	—	2
pen or pencil	1	2
poter board, white	1 sheet	4
string	—	4
string (or fishing line)	3 meters	5
string (or twine)	1 meter	1
tape or glue	—	4, 5
tennis ball	1	1

Nonconsumable Materials

Item	Quantity per lab group	Activity
bicycle	1	2
bolt, 4-cm length	1	1
bowling ball	1	5
metric ruler	1	3
meter stick or tape measure	1	1
phonograph turntable, 33 1/3 rpm and 45 rpm	1	3
screwdriver	1	1
stopwatch	1	4, 5
tape measure	1	2

Key to activities:

1. David and Goliath
2. Bicycle Revolution
3. Roundabouts
4. Points of View
5. Something New?

Activity Sources

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- Hewitt, P. *Conceptual Physics*, 7th ed. New York: Harper Collins College Publishers, 1993.
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